Balkan MO 2006

Nicosia, Cyprus

1 Let a, b, c be positive real numbers. Prove the inequality

$$\frac{1}{a(b+1)}+\frac{1}{b(c+1)}+\frac{1}{c(a+1)}\geq\frac{3}{1+abc}.$$

2 Let ABC be a triangle and m a line which intersects the sides AB and AC at interior points D and F, respectively, and intersects the line BC at a point E such that C lies between B and E. The parallel lines from the points A, B, C to the line m intersect the circumcircle of triangle ABC at the points A_1, B_1 and C_1, respectively (apart from A, B, C). Prove that the lines A_1E , B_1F and C_1D pass through the same point.

Greece

3 Find all triplets of positive rational numbers (m,n,p) such that the numbers $m+\frac{1}{np}$, $n+\frac{1}{pm}$, $p+\frac{1}{mn}$ are integers.

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4 Let m be a positive integer and $\{a_n\}_{n\geq 0}$ be a sequence given by $a_0=a\in\mathbb{N}$, and

$$a_{n+1}=\begin{cases}\frac{a_n}{2} & \text{if } a_n\equiv 0 \pmod 2,\\ a_n+m & \text{otherwise.}\end{cases}$$

Find all values of a such that the sequence is periodical (starting from the beginning).

